МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
имени М.В. ЛОМОНОСОВА

ОЛИМПИАДНАЯ РАБОТА

Наименование олимпиады школьников: **«Покори ВоробьевыГоры!»**

Профиль олимпиады: **ФИЗИКА**

ФИО участника олимпиады: **Пламадяла Дмитрий Романович**

Класс: 11

Технический балл: **81**

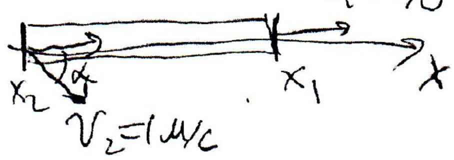
1	2	3	4	Σ	итог
5	5	5	2	17	81
16	12	19	17	64	

Дата проведения: **26 марта 2021 года**

Чистовик

№1

1. Вопрос



Если Ox – вдоль стержня, x_1 и x_2 – концы стержня,

Условие ~~нер~~ жёсткости стержня → т.к. $x_2-x_1=const$

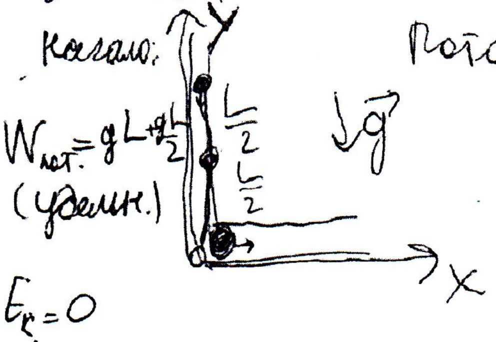
$v_{2x}-v_{1x}=0 \rightarrow v_{2x}=v_{1x}$

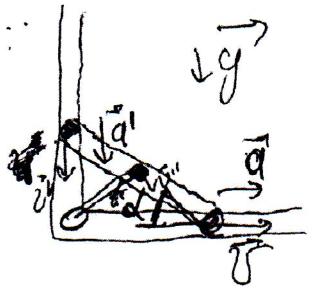
$v_2\cos\alpha=v_1$

$\cos\alpha=\frac{v_1}{v_2}=\frac{1}{2}$

$\boxed{\alpha=60^\circ}$

Задача





a, v – ускор. и скор. нижнего,

a', v' – // – верхнего

~~[illegible]~~

a'', v'' – среднего

Усл. жёсткости: $x_1'-x_2'=const$

$v_{x'}=v'_{x'}$

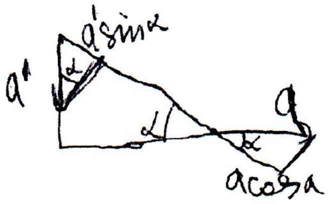
$v\cos\alpha=v'\sin\alpha$

$v'=v\,ctg\,\alpha$

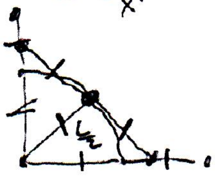
$\frac{d}{dt}$

$a_{x'}=a'_{x'}$ $\quad a\cos\alpha=a'\sin\alpha$

$a'=a\,ctg\,\alpha$



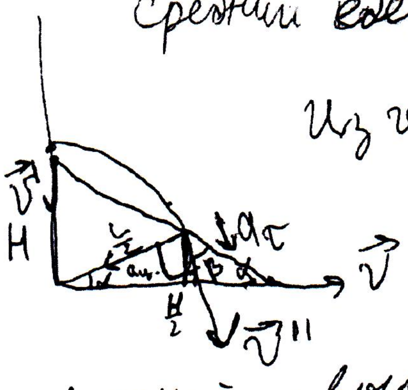
$v''_{x'}=v_{x'}$ – тоже из жёсткости



средний едет по окружности – медиана прямоуг. $\triangle$ $\frac{L}{2}$

Из геометрии: $2\alpha+90^\circ+\beta=180^\circ$ (равноб. $\triangle$)

$\beta=90^\circ-2\alpha$



$v''_{x'}=v''\cos\beta=v\cos\alpha$

$v''=v\frac{\cos\alpha}{\sin2\alpha}=\frac{v}{2\sin\alpha}$

Если верхний на высоте $H=L\sin\alpha$, то средний – на выс. $\frac{H}{2}=\frac{L}{2}\sin\alpha$

1) ~~Тогда~~ ЗСЭ (трения нет): (удельн., масса одинак.) $\Rightarrow$ $\Delta W_{пот.}+\Delta E_k=0$

$$\left(gH+g\frac{H}{2}\right)-\left(gL+g\frac{L}{2}\right)+\frac{v^2}{2}+\frac{v^2}{2tg^2\alpha}+\cancel{\frac{v^2\cos^2\alpha}{[illegible]}}\ \frac{v^2}{8\sin^2\alpha}=0$$

Чистовик (№2

1.(прод.) $v^2\left(\underbrace{1+\frac{1}{tg^2\alpha}}_{\frac{1}{\sin^2\alpha}}+\frac{1}{4\sin^2\alpha}\right)=3g(L-H)=3gL(1-\sin\alpha)$

$$v^2=\frac{3gL(1-\sin\alpha)}{\frac{5}{4\sin^2\alpha}}=\frac{12gL}{5}\sin^2\alpha(1-\sin\alpha)$$

$$\boxed{v=\sqrt{\frac{12}{5}gL\sin^2\alpha(1-\sin\alpha)}}$$

$$v(60^\circ)=\sqrt{\frac{12}{5}\cdot10\cdot0{,}8\cdot\frac{3}{4}\left(1-\frac{\sqrt3}{2}\right)}=\sqrt{4\cdot9\cdot\frac{2}{5}\left(1-\frac{\sqrt3}{4}\right)}=6\sqrt{\frac{2}{5}\left(1-\frac{\sqrt3}{2}\right)}=6\sqrt{0{,}54}\approx4{,}4\ \text{м/с}$$

$\sqrt3\approx1{,}73$ $\qquad\sqrt{0{,}54}\approx0{,}73$

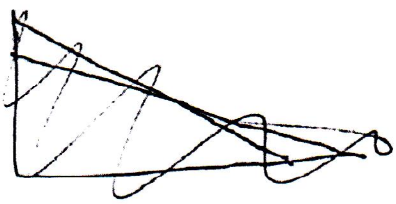
$1-\frac{\sqrt3}{2}\approx0{,}135$

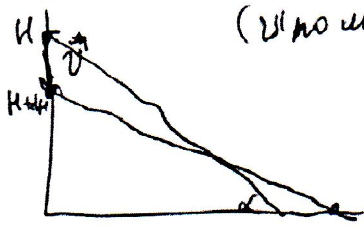
2) $v=\sqrt{\frac{12}{5}gL}\sqrt{1-\sin\alpha}\cdot\sin\alpha$

~~За dt~~ $\qquad \sin\alpha=\frac{H}{L}$ $\qquad \frac{d\sin\alpha}{dt}=\frac{dH}{Ldt}=\frac{-v'}{L}=-\frac{v}{Ltg\alpha}$ $\qquad a=\frac{dv}{dt}=\frac{dv}{d\sin\alpha}\cdot\frac{d\sin\alpha}{dt}$

(v' по мод.)

$\cancel{\frac{dv}{d\sin\alpha}=\frac{v}{\sin\alpha}+\frac{-v}{\sqrt{1-\sin\alpha}}}$





$\cancel{a=\frac{v^2}{Ltg\alpha}\left(\frac{1}{\sqrt{1-\sin\alpha}}-\frac{1}{\sin\alpha}\right)=}$

$\cancel{\frac{12g\cos\alpha}{5}\left(\sqrt{1-\sin\alpha}-\sin\alpha\right)}=\cancel{\frac{12g\cos\alpha\sin\alpha}{5\sin\alpha}\left(\sin\alpha-\sqrt{1-\sin\alpha}\right)\sqrt{1-\sin\alpha}}$

$$\frac{dv}{d\sin\alpha}=\sqrt{\frac{12}{5}gL}\left[\sqrt{1-\sin\alpha}-\frac{\sin\alpha}{2\sqrt{1-\sin\alpha}}\right]=\left\{\frac{v}{\sin\alpha}-\frac{v}{2(1-\sin\alpha)}\right\}$$

$$a=\frac{v^2\cos\alpha}{L\sin\alpha}\left(-\frac{1}{\sin\alpha}+\frac{1}{2(1-\sin\alpha)}\right)=\frac{12}{5}g\sin\alpha\cos\alpha\left(-1+\sin\alpha+\frac{\sin\alpha}{2}\right)=\frac{12}{5}g\sin2\alpha(3\sin\alpha-2)$$

$$\boxed{a=\frac{12}{5}\sin2\alpha(3\sin\alpha-2)g}$$

$$a(60^\circ)=\frac{12}{5}\frac{\sqrt3}{2}\left(3\frac{\sqrt3}{2}-2\right)\cdot10=\frac{3}{5}\sqrt3(3\sqrt3-4)\cdot10=10\cdot\frac{27-12\sqrt3}{5}\approx\boxed{12{,}5\ \frac{\text{м}}{\text{с}^2}}$$

$12\sqrt3\approx20{,}76$

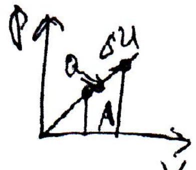
$27-12\sqrt3=6{,}24$

$$a=12{,}5\ \frac{\text{м}}{\text{с}^2}$$

Чистовик

N 3

2. Вопрос: $p=\alpha V$ $\quad A=1$ Дж



$\downarrow$ $\quad Q = A + \Delta U$

ур-е политр. $pV^{\gamma} = \alpha \quad \gamma = -1$

$i = 3 \quad \frac{C - C_V}{C - C_p} = -1 \to 2C - 3R = -2C + 5R \to 4C = 8R$

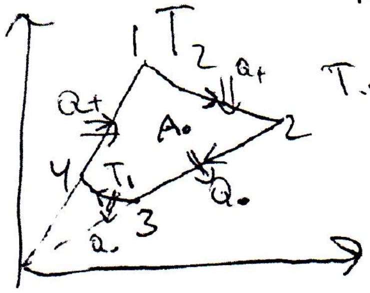
$C = 2R$

$Q = C\nu\Delta T \quad \Delta U = C_V \nu \Delta T$

$A = \nu(C - C_V)\Delta T \to \Delta T = \frac{A}{\nu(2R - \frac{3}{2}R)} = \frac{2A}{\nu R} \quad A = \frac{\nu}{2}R\Delta T$

$\boxed{\Delta U = \frac{3}{2}R\nu \cdot \frac{2A}{\nu R} = 3A} \quad Q = 4A$

Задача. $i = 3$, ν = const $\quad T_2 = nT_1$ (т.к. на $p=\alpha V$ $\quad T \sim V^2 \sim p^2$ и там Тмакс. при p, V макс.)



Т. маш. $\downarrow$ $A_0 > 0$

$\eta = \frac{A_0}{Q_+}$ - ? $\quad \eta = 1 - \frac{|Q_-|}{Q_+}$

$A_{12} = -kA_{23} \quad \Delta U_{23} = \frac{3}{2}\nu R(T_1 - T_2) = 3A_{23}$ - док. выше.

$A_{23} = \frac{\nu R(T_1 - T_2)}{2} = -\nu RT_1$

n=2

k=2

$Q_{12} = A_{12} > 0 \quad Q_+ = Q_{41} + Q_{12} \quad Q_{41} = 4A_{41} = \frac{4}{3}\Delta U_{41} = \frac{4}{3}\cdot\frac{3}{2}\nu R(T_2 - T_1) =$

$= -\frac{4}{3}\Delta U_{23} = 4A_{23}$

$Q_+ = -(2+k)A_{23}$

$|Q_-| = |Q_{23} + Q_{34}| \quad Q_{34} = A_{34} \le 0$

$|Q_-| = -A_{34} - 4A_{23} \quad Q_{23} = 4A_{23} < 0$

$\frac{p_2}{p_3} = \frac{V_2}{V_3} \quad \frac{p_1}{p_4} = \frac{V_1}{V_4} \quad p_1V_1 = p_2V_2 = \frac{T_2}{\nu R}$

$p_3V_3 = p_4V_4 = \frac{T_1}{\nu R}$

$\to p_1 = \frac{p_4}{V_4}$

$p_4 = \frac{p_1}{V_1}V_4 \to \frac{T_1}{\nu RV_4}\cdot\frac{\nu RV_1}{2T_1} = \frac{V_4}{V_1}$

$V_1^2 = 2V_4^2$

$p_3 = \frac{p_2}{V_2}V_3 \to V_2^2 = 2V_3^2$

$p_1 = \frac{2T_1}{\nu RV_1} \quad p_2 = \frac{2T_1}{\nu RV_2} \quad p_3 = \frac{T_1}{\nu RV_3} \quad p_4 = \frac{T_1}{\nu RV_4}$

$p_1 = \sqrt{2}p_4 \quad p_2 = \sqrt{2}p_3$

$V_1 = \sqrt{2}V_4 \quad V_2 = \sqrt{2}V_3$

Чистовик.

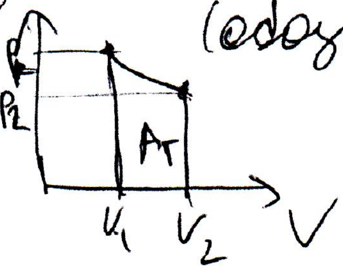
№ 8

№ 4.

2. (прод.) (обозн. другие)

Вс некая задача:

$A_{T=const}$



$$pV = \nu RT = const$$

$$p = \frac{\nu RT}{V} = \frac{\alpha}{V} \qquad \delta A = p\,dV = \frac{\alpha}{V}dV$$

$$A_T = \int_{V_1}^{V_2} \frac{\alpha}{V}dV = \alpha \ln\frac{V_2}{V_1} = \nu RT \ln\frac{V_2}{T_1} \qquad \alpha = \nu RT$$

$$A_{12} = 2\nu RT_1 \ln\frac{V_2}{V_1} = -k(-\nu RT_1) = k\nu RT_1 \rightarrow 2\ln\frac{V_2}{V_1} = k$$

$$\frac{V_2}{V_1} = e^{\frac{k}{2}} = e$$

$$A_{34} = \nu RT_1 \ln\frac{V_4}{V_3} = \nu RT_1(-1) = -\nu RT_1$$

$$\sqrt{2}V_3 = V_2 = eV_1 = e\sqrt{2}V_4$$

$$V_3 = eV_4$$

$$\cancel{\eta = 1 - \frac{\nu RT_1 + 4\nu RT_1}{(2+k)\nu RT_1} = 1 - \frac{5}{4} =}$$

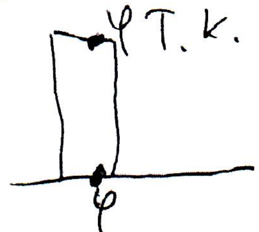
$$A_0 = A_{12} + A_{23} + A_{34} + A_{41} = k\nu RT_1 - \nu RT_1 - \nu RT_1 + \nu RT_1 = \nu RT_1$$

$$\eta = \frac{A_0}{Q_+} = \frac{\nu RT_1}{(2+k)\nu RT_1} = \boxed{\text{ответ: } \frac{1}{4} = \eta}$$

Черновик

№5.

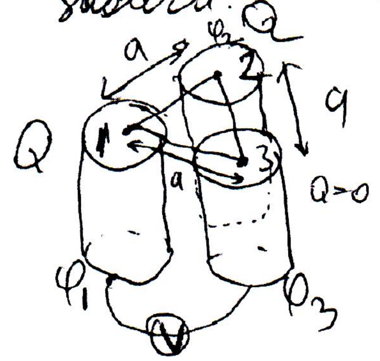
3. вопрос:

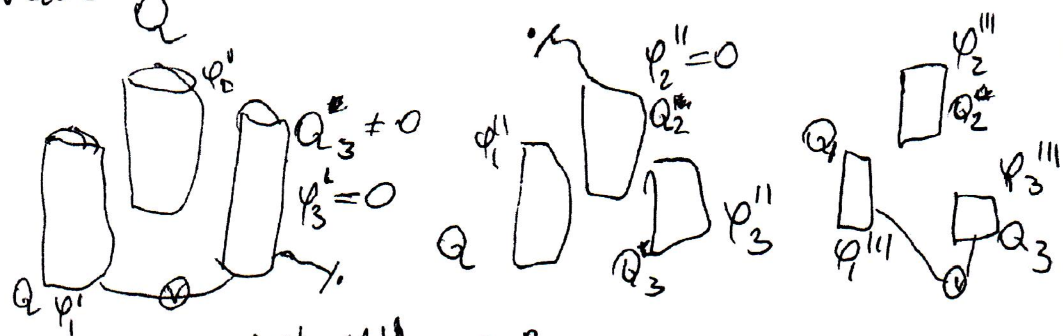


φ Т.к. он проводник, $\Delta\varphi = 0$ — по всей пов-ти, в т.ч. в ед. осн.

(пусть $Q>0$ — это никак не вл.)

Задача:





$$U' = |\varphi_3' - \varphi_1'| = 60В$$

$$|\varphi_3 - \varphi_1| = U = 40В$$

$U''' - ?$
$= |\varphi_3''' - \varphi_1'''|$

Т.к. геометрия не мен., ~~в любой~~ при люб. распр. зар. в силу суперпозиции зависит только от зарядов линейно:

$$\varphi_1 = \alpha_{11} q_1 + \alpha_{12} q_2 + \alpha_{13} q_3$$
$$\varphi_2 = \alpha_{21} q_1 + \alpha_{22} q_2 + \alpha_{23} q_3$$
$$\varphi_3 = \alpha_{31} q_1 + \alpha_{32} q_2 + \alpha_{33} q_3$$

потенц. 1, 2 и 3-го цил. соотв. (они равны по всей пов-ти — проводник)

в силу сим. $\cancel{\alpha_{ij} = \alpha_{ji}}$ ($\alpha, \beta > 0$, т.к. $+q \uparrow \varphi$)

$\alpha_{11} = \alpha_{22} = \alpha_{33} = \alpha$

$\alpha_{12} = \alpha_{21} = \ldots = \alpha_{32} = \beta$

$\cancel{\alpha_{12} = \alpha_{21}}$

при заземл. $\varphi_i = 0$, q_i меняется

0) до заз:

$$\varphi_1 = \alpha Q + \beta Q + 0$$
$$\varphi_2 = \beta Q + \alpha Q + 0$$
$$\varphi_3 = \beta Q + \beta Q + 0$$

$\rightarrow \varphi_1 = \varphi_2 = (\alpha + \beta) Q = \varphi$

$\varphi_3 = 2\beta Q$

$U = |(\beta - \alpha)| Q$ ~~(можно предп., что $\alpha > \beta$, т.к. на себя близкого влияние больше)~~

1) заземл. I: $\varphi_1' = \alpha Q + \beta Q + \beta Q_3 = \varphi + \beta Q_3$

Чистовик

№8.

3.(прод.) $\varphi_2' = \beta Q + \alpha Q + \beta Q_3 = \varphi + \beta Q_3 = \varphi_1'$

$0 = \varphi_3' = 2\beta Q + \alpha Q_3 = \varphi_3 + \alpha Q_3 \Rightarrow Q_3 = \frac{-2\beta}{\alpha} Q$

$U' = |\varphi_3' - \varphi_1'| = |\alpha Q_3 + \beta Q - \alpha Q - \beta Q_3| = |(\alpha - \beta)(Q_3 - Q)| = U\frac{|Q_3 - Q|}{Q} = U\left|\frac{2\beta - \alpha}{\alpha}\right|$

$\left|\frac{2\beta}{\alpha} - 1\right| = \frac{U'}{U} \qquad \frac{2\beta + \alpha}{\alpha} = \frac{U'}{U} \to \beta = \frac{\alpha}{2}\left(-1 + \frac{U'}{U}\right)$ ~~$(\beta = \frac{?}{?}\alpha)$~~

$\beta = \frac{\alpha}{4} < \alpha$

$U = (\alpha - \beta) Q = \frac{3}{4}\alpha Q$ ~~$\Rightarrow Q$~~ $\alpha Q = \frac{4}{3} U$

$Q_3 = -\frac{Q}{2}$

2) после II заз:

$\varphi_2'' = 0 = \beta Q + \alpha Q_2 + \beta Q_3 \Rightarrow Q_2 = \frac{-\beta}{\alpha}(Q + Q_3) = -\frac{\beta}{\alpha}\frac{Q}{2} = -\frac{Q}{8}$

3) после III заз: $\varphi_1''' = 0 = \alpha Q_1 + \beta Q_2 + \beta Q_3$

$Q_1 = \frac{-\beta}{\alpha}(Q_2 + Q_3) = \frac{5}{32} Q$

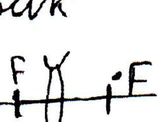
~~U'''~~ $\varphi_3''' = \beta Q_1 + \beta Q_2 + \alpha Q_3 = \alpha Q\left[\frac{1}{4}\frac{5}{32} + \frac{1}{4}\left(-\frac{1}{8}\right) + 1\cdot\left(-\frac{1}{2}\right)\right] =$

$= \frac{4}{3} U\left[\frac{5}{2^7} - \frac{1}{2} - \frac{1}{2^5}\right] = \frac{4}{3} U \frac{5 - 2^6 - 2^2}{2^7} = \frac{U\cdot(-63)}{3\cdot 2^5} = -\frac{21}{32} U$

$U''' = |\varphi_3''' - 0| = \frac{21}{32} U = \frac{21}{32}\cdot 2^3\cdot 5 = \frac{21\cdot 5}{4} = \boxed{26{,}25\ В}$

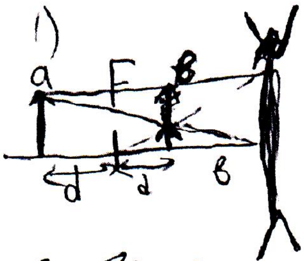
Ответ: 26,25 В

Черновик №7.

4. В: $DF = 1$  $D = \cancel{\frac{1}{F}} = \frac{1}{b} + \frac{1}{a}$ (a - до предм.; b - расст. до изобр.) отриц., если мнимое

(Если считать $F<0$ у рассеив.)

3: 1)



$a = |b| + 2d$ $\quad a = 2d - b = d - F$

мнимое изобр. $b<0$, $F<0$ $\quad |F| = |b| + d$

$F = b - d$

$b = F + d$

$D = a^{-1} + b^{-1}$

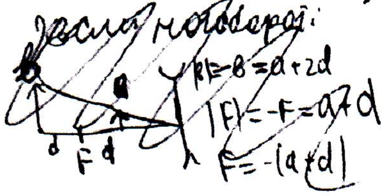
$\frac{1}{F} = \frac{1}{d-F} + \frac{1}{d+F} \to d^2 - F^2 = dF + F^2 + dF - F^2$

$F^2 + 2dF - d^2 = 0$

$F = -d \pm \sqrt{d^2 + d^2} = -d \pm \sqrt{2}d$

т.к. $F<0$ $\quad F_1 = -(1+\sqrt{2})d$

$F_2 = (-1+\sqrt{2})d$?



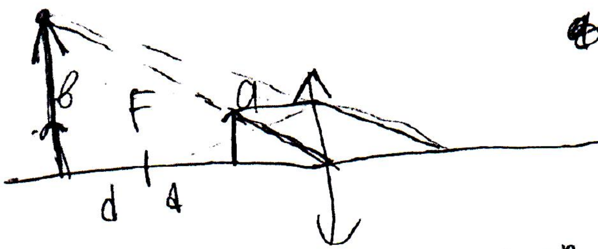
$F_1 = -(1+\sqrt{2})d$

$F_1 = -(1+\sqrt{2})\cdot 0{,}15 = -2{,}4\cdot 0{,}15 = -\frac{24\cdot 15}{1000} = -\frac{360}{1000}$

$\boxed{D_1 = \frac{1}{F_1} \approx -\frac{100}{36} = -2{,}8}$ $\quad D_1 = -\frac{1}{(1+\sqrt{2})d}$

~~$|b| = a+2d$ $\quad a = 2d - b$ $\quad |F| = a+d$ $\quad d - F = a$~~

2)



$F>0$ $\quad b<0$ - мнимое изобр.

$F = a+d$ $\quad a = F-d$ | $-b = a + 2d = F + d$

$b = -(F+d)$

$\frac{1}{F} = \frac{1}{F-d} + \frac{-1}{F+d}$

$F^2 - d^2 = F^2 + Fd - F^2 + Fd$ - то же, что и сверху $F_2 = (-1+\sqrt{2})d$

$D_2 = \frac{1}{F_2} = \frac{1}{(\sqrt{2}-1)d}$

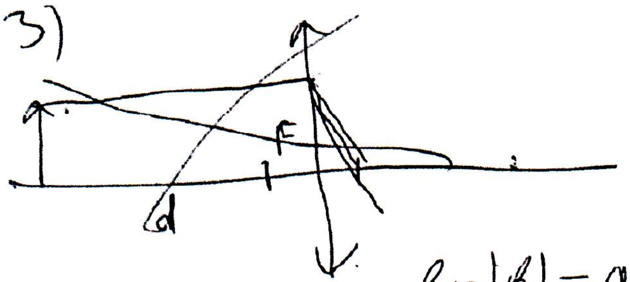
$D_2 = \frac{1}{0{,}4\cdot 0{,}15} = \frac{5\cdot 100}{2\cdot 15} = \frac{100}{6} = 17$ Дптр

~~Чистовик~~ ~~Черновик~~ Чистовик

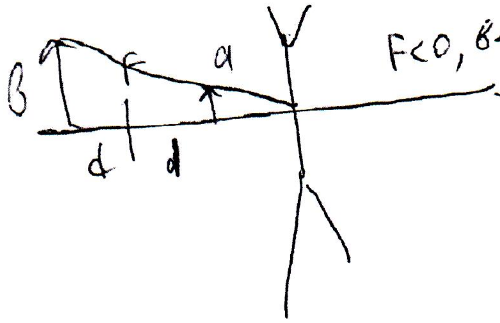
$\boxed{N 8}$

4. (прод.)

3)



$-b = |b| = a+2d$ → ~~[illegible]~~



$F<0,\ b<0$

$-F = a+d$ → ~~[illegible]~~

$a = -F-d$

$b = F-d$

$\frac{1}{F} = \frac{-1}{F+d} + \frac{1}{F-d}$ → тоже ур-ие, что и в 2) $\Rightarrow$ совп. с F_1

$\boxed{\text{Ответ: } D = \frac{1}{(-1\pm\sqrt{2})d}}$

~~Чистовик~~ Черновик

N 2

1. ~~(прод.)~~ $\frac{v^2}{2}\left(1+ctg^2\alpha+\frac{cosec^2\alpha}{4}\right)=g\cdot\frac{3}{2}(L-H)$ $\cdot\sin^2\alpha$

$v^2=\frac{3}{2}g(L-H)\frac{\sin^2\alpha}{(\sin^2\alpha+\cos^2\alpha+1)}$ 2

$\sin 3\alpha=\sin\alpha(1-2\sin^2\alpha)-2\sin\alpha\cos\alpha(1-\sin$

$=\sin\alpha-2\sin\alpha\ldots$

$3\sin\alpha-4\sin^3\alpha$

$v^2=\frac{3g}{2}(L-H)\sin^2\alpha$

$v=\sqrt{\frac{3}{2}g(L-L\sin\alpha)}\,\sin\alpha$

$\boxed{v=\sqrt{\frac{3}{2}gL(1-\sin\alpha)}\,\sin\alpha}$

$v(60^\circ)=\sqrt{\frac{3}{2}\cdot10\cdot0{,}8\left(1-\frac{\sqrt3}{2}\right)}\,\frac{\sqrt3}{2}$ $17^2=289$

$1-0{,}855=0{,}135$

$v'=\sqrt{3gL(1-\sin\alpha)}\,\cos\alpha$

v_0

$v=v_0\sin\alpha$

$3{,}00 \,|\, 3$; $1{,}73$; $2\,2\,0\,0$; $1\,8\,9$; $343\,|\,1100$; 1029 ; 71

2) $a=\frac{dv}{dt}=\frac{dv}{d\sin\alpha}\,\frac{d\sin\alpha}{dt}$

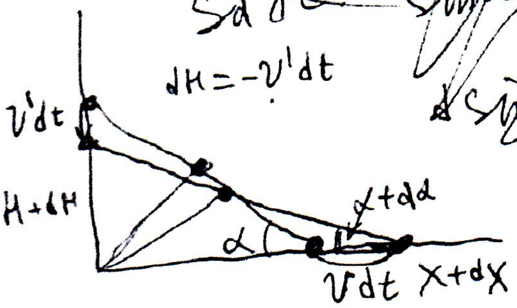
~~$\frac{H+dH}{X+dx}=\frac{H-v'dt}{X+vdt}$~~   ~~$tg\,\alpha=\frac{H}{X}$~~

$0{,}4\ \ 0{,}35=0{,}54$

~~$3a\,dt$~~ ~~$\sin\alpha+d\alpha$~~

$dH=-v'dt$

~~$d\sin\alpha=\frac{H-v_0\cos\alpha\,dt}{X+v_0\sin\alpha\,dt}$~~ $-\frac{H}{X}=\frac{xH-v_0\cos\alpha\,x\,dt-Hx-Hv_0\sin\alpha\,dt}{X(x+v_0\sin\alpha\,dt)}=$ 0



$=-\frac{v_0\,dt}{x^2}(x\cos\alpha+H\sin\alpha)$

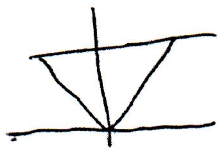
$54 \,|\, 2$; 173 ; 49 ; 500 ; 1435 ; 29 ; 465 ; 5

$d\sin\alpha=\frac{dH}{L}=\frac{-v'dt}{L}=\frac{-v\,dt}{L\,tg\alpha}=-\frac{v_0\cos\alpha\,dt}{L}$

$3a\,dt\cdot\sin(\alpha+d\alpha)=\frac{H+dH}{L}$

$a=\frac{dv}{d\sin\alpha}=\frac{d(v_0\sin\alpha)}{d\sin\alpha}=v_0+\sin\alpha\frac{dv_0}{d\sin\alpha}=v_0+\sin\alpha\frac{d\sqrt{3gL(1-\sin\alpha)}}{d\sin\alpha}$

$\frac{dv_0}{d\sin\alpha}=\sqrt{3gL}\,\frac{d\sqrt{1-\sin\alpha}}{d\sin\alpha}=\sqrt{3gL}\,\frac{-1}{2\sqrt{1-\sin\alpha}}=-\frac{1\,v_0}{2(1-\sin\alpha)}$



$a=\left(v_0-\frac{v_0\sin\alpha}{2(1-\sin\alpha)}\right)\left(-\frac{v_0\cos\alpha}{L}\right)=\frac{v_0^2\cos\alpha}{L}\left(\frac{2-2\sin\alpha-2+2\sin\alpha}{2(1-\sin\alpha)}\right)=$

$=\frac{v_0^2\cos\alpha}{L}\cdot\frac{3\sin\alpha-2}{2(1-\sin\alpha)}=\frac{3gL(1-\sin\alpha)}{2L(1-\sin\alpha)}(3\sin\alpha-2)=\frac{3}{2}g(3\sin\alpha-2)$

$a=$

~~Чистовик~~ Черновик

№ 2.

2. (прод.)

$$\cancel{2A_{41} = -4A_{23}}$$

$$\cancel{A_{41} = \frac{p_1+p_4}{2}(V_1-V_4) \quad A_{23} = \frac{p_2+p_3}{2}(V_3-V_2)}$$