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(123.8)

МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
имени М.В.ЛОМОНОСОВА

Вариант С-3

дешифр

Место проведения Москва
город

ПИСЬМЕННАЯ РАБОТА

Олимпиада школьников "Покори Воробьевы горы!"
наменование олимпиады

по математике
профиль олимпиады

Асташкиной Анны Дмитриевны
фамилия, имя, отчество участника (в родительном падеже)

Шифр	Сумма	1	2	3	4	5	6	7	8
80-76-86-37	85	20	20	20	20	5	0		

80-76-86-37 (123.8)

Черновик

$$1+\sqrt{2}\sin x(\cos x-2\sin x)+\sqrt{2}\cos x(2\cos x+\sin x)=2\cos^2\left(x+\frac{\pi}{8}\right)$$

$$2\cos^2\left(x+\frac{\pi}{8}\right)=2\cdot\frac{1+\cos\left(2x+\frac{\pi}{4}\right)}{2}=1+\cos\left(2x+\frac{\pi}{4}\right)$$

$$\sqrt{2}\sin x(\cos x-2\sin x)+\sqrt{2}\cos x(2\cos x+\sin x)=\cos\left(2x+\frac{\pi}{4}\right)$$

$$\sqrt{2}\sin x(\cos x-2\sin x)+\sqrt{2}\cos x(2\cos x+\sin x)=\cos 2x\cos\frac{\pi}{4}-$$
$$-\sin 2x\sin\frac{\pi}{4}$$

$$\sqrt{2}\sin x\cos x-2\sqrt{2}\sin^2 x+2\sqrt{2}\cos^2 x+\sqrt{2}\sin x\cos x=$$
$$=\frac{\sqrt{2}}{2}(\cos^2 x-\sin^2 x)-\frac{\sqrt{2}}{2}\cdot 2\sin x\cos x$$

$$2\sqrt{2}\sin x\cos x+2\sqrt{2}(\cos^2 x-\sin^2 x)=\frac{\sqrt{2}}{2}(\cos^2 x-\sin^2 x)-\sqrt{2}\sin x\cos x$$

$\cancel{3\sqrt{2}\sin x\cos x}$ $4\sqrt{2}\sin x\cos x+4\sqrt{2}(\cos^2 x-\sin^2 x)=\sqrt{2}(\cos^2 x-\sin^2 x)-2\sqrt{2}\sin x\cos x$

$\cancel{\cos^2\alpha=2\cos}\ 2\cos^2\frac{\alpha}{2}$

$\cos 2x=2\cos^2\alpha-1$

$2\cos^2\alpha=1+\cos 2\alpha$

$\cos^2\alpha=\frac{1+\cos 2\alpha}{2}$

$$6\sqrt{2}\sin x\cos x+3\sqrt{2}(\cos^2 x-\sin^2 x)=0$$

$$2\sin x\cos x+(\cos^2 x-\sin^2 x)=0$$

$$\sin 2x+\cos 2x=0$$

$$\sqrt{2}\left(\underbrace{\frac{1}{\sqrt{2}}}_{\cos\frac{\pi}{4}}\sin 2x+\underbrace{\frac{1}{\sqrt{2}}}_{\sin\frac{\pi}{4}}\cos 2x\right)=0$$

$$\sin\left(2x+\frac{\pi}{4}\right)=0$$

	1	-6	7	-1
1	1	-5	2	

$$(x-x_1)(x-x_2)(x-x_3)=0$$

$$(x^2-x_1x-x_2x+x_1x_2)(x-x_3)=0$$

$$x^3-x_1x^2-x_2x^2+x_1x_2x-x_3x^2+x_3x_1x+$$
$$+x_2x_3x-x_1x_2x_3=0$$

$$\begin{cases}-x_1x_2x_3=-1\\ x_1x_2+x_2x_3+x_1x_3=7\\ -(x_1+x_2+x_3)=-6\end{cases}$$

$$x^3+x^2(-x_1-x_2-x_3)+x(x_1x_2+x_2x_3+x_1x_3)-x_1x_2x_3$$

№1 Чистовик

$1+\sqrt{2}\sin x(\cos x-2\sin x)+\sqrt{2}\cos x(2\cos x+\sin x)=2\cdot\cos^2(x+\frac{\pi}{8}) \Leftrightarrow$

$1+\sqrt{2}\sin x\cos x-2\sqrt{2}\sin^2 x+2\sqrt{2}\cos^2 x+\sqrt{2}\sin x\cos x=2\cdot\frac{1+\cos(2x+\frac{\pi}{4})}{2} \Leftrightarrow$

$1+2\sqrt{2}\sin x\cos x+2\sqrt{2}(\cos^2 x-\sin^2 x)=1+\cos 2x\cdot\frac{\sqrt{2}}{2}-\sin 2x\cdot\frac{\sqrt{2}}{2} \Leftrightarrow$

$2\sqrt{2}\sin x\cos x+2\sqrt{2}(\cos^2 x-\sin^2 x)=\frac{\sqrt{2}}{2}(\cos^2 x-\sin^2 x)-\sqrt{2}\sin x\cos x$

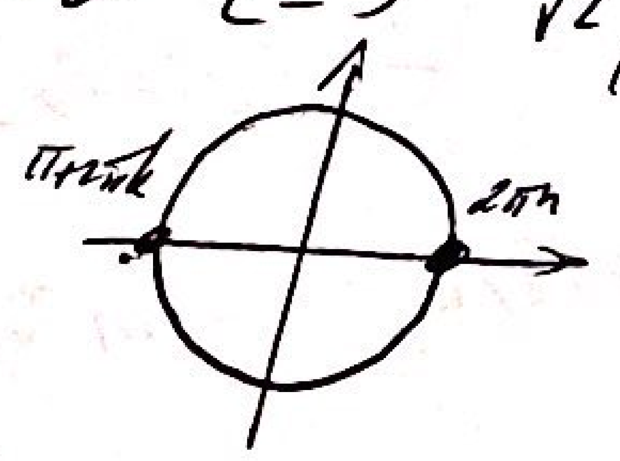
$4\sqrt{2}\sin x\cos x+4\sqrt{2}(\cos^2 x-\sin^2 x)=\sqrt{2}(\cos^2 x-\sin^2 x)-2\sqrt{2}\sin x\cos x$

$6\sqrt{2}\sin x\cos x+3\sqrt{2}(\cos^2 x-\sin^2 x)=0$

$2\sin x\cos x+(\cos^2 x-\sin^2 x)=0$

$\sin 2x+\cos 2x=0 \Leftrightarrow \sqrt{2}\left(\overset{\cos\frac{\pi}{4}}{\frac{\sqrt{2}}{2}}\sin 2x+\overset{\sin\frac{\pi}{4}}{\frac{\sqrt{2}}{2}}\cos 2x\right)=0$

$\sin(2x+\frac{\pi}{4})=0$



$2x+\frac{\pi}{4}=\pi n, n\in Z$

$2x=-\frac{\pi}{4}+\pi n, n\in Z$

$x=-\frac{\pi}{8}+\frac{\pi n}{2}, n\in Z$

Ответ: $\{-\frac{\pi}{8}+\frac{\pi n}{2}\}, n\in Z$

Черновик
$$\begin{cases} x_1x_2x_3=1 \\ x_1x_2+x_2x_3+x_1x_3=7 \\ x_1+x_2+x_3=6 \end{cases}$$

$$\begin{cases} (x_1+x_2)(x_2+x_3)(x_3+x_1)=-c \quad ② \\ (x_1+x_2)(x_2+x_3)+(x_1+x_2)(x_3+x_1)+(x_2+x_3)(x_3+x_1)=b \quad ③ \\ x_1+x_2+x_2+x_3+x_3+x_1=-a \quad ① \end{cases}$$

① $2(x_1+x_2+x_3)=-a$

$2\cdot 6=-a \quad a=-12$

③ $x_1x_2+x_2^2+x_1x_3+x_2x_3+x_1x_3+x_2x_3+x_1^2+x_2x_1+$
$+x_2x_3+x_3^2+x_1x_2+x_1x_3=b$

$x_1^2+x_2^2+x_3^2+3x_1x_2+3x_1x_3+3x_2x_3=b$

$(x_1+x_2+x_3)^2+x_1x_2+x_2x_3+x_1x_3=b$

$36+7=b \quad b=43$

② $(x_1x_2+x_2^2+x_1x_3+x_2x_3)(x_3+x_1)=-c$

$\cancel{x_1x_2x_3}+\cancel{x_2^2x_3}+\cancel{x_1x_3^2}+\cancel{x_2x_3^2}+x_1^2x_2+x_2^2x_1+\cancel{x_1^2x_3}+\cancel{x_2x_3x_1}=$

$=x_2x_3(x_1+x_2+x_3)+x_1x_3(x_1+x_2+x_3)+x_1x_2(x_1+x_2+x_3)-$

$-x_1x_2x_3=-c$

$(x_1+x_2+x_3)(x_1x_2+x_2x_3+x_1x_3)-x_1x_2x_3=-c$

Чистовик

№3 x_1, x_2, x_3 - корни $x^3-6x^2+7x-1=0$ => по т. Виета

$$\begin{cases} x_1+x_2+x_3=6 \\ x_1x_2+x_2x_3+x_1x_3=7 \\ x_1x_2x_3=1 \end{cases}$$

(x_1+x_2); (x_2+x_3); (x_3+x_1) - корни $x^3+ax^2+bx+c=0$ =>

по т. Виета:

$$\begin{cases} x_1+x_2+x_2+x_3+x_3+x_1=-a \quad 1 \\ (x_1+x_2)(x_2+x_3)+(x_1+x_2)(x_3+x_1)+(x_2+x_3)(x_3+x_1)=b \quad 2 \\ (x_1+x_2)(x_2+x_3)(x_1+x_3)=-c \quad 3 \end{cases}$$

Рассмотрим 1

$2(x_1+x_2+x_3)=-a \quad 2\cdot 6=-a \quad \underline{a=-12}$

Рассмотрим 2

$x_1x_2+x_2^2+x_1x_3+x_2x_3+x_1x_3+x_2x_3+x_1^2+x_1x_2+x_2x_3+x_3^2+x_1x_2+x_1x_3=b$

$x_1^2+x_2^2+x_3^2+3x_1x_2+3x_2x_3+3x_1x_3=b$

$(x_1+x_2+x_3)^2+x_1x_2+x_2x_3+x_1x_3=b$

$36+7=b \quad \underline{b=43}$

Рассмотрим 3

$(x_1x_2+x_2^2+x_1x_3+x_2x_3)(x_1+x_3)=-c$

$\underline{x_1^2x_2}+\underline{x_2^2x_1}+\underline{x_1^2x_3}+\underline{x_1x_2x_3}+\underline{x_1x_2x_3}+x_2^2x_3+\underline{x_1x_3^2}+x_2x_3^2=-c$

$x_1x_2(x_1+x_2+x_3)+x_1x_3(x_1+x_2+x_3)+x_2x_3(x_1+x_2+x_3)-x_1x_2x_3=-c$

$(x_1+x_2+x_3)(x_1x_2+x_1x_3+x_2x_3)-x_1x_2x_3=-c$

$6\cdot 7-1=-c \quad 41=-c \quad \underline{c=-41}$

Ответ: при $a=-12$, $b=43$, $c=-41$

Черновик

A — 1 — B $V_A = 2V_B$

1) в 14:00 : велосипедист ост: велосипедист

$t_a = \frac{1}{V_A} = \frac{1}{2V_B}$ $t_B = \frac{1}{V_B} + 2$

$t_a = t_B - 1$

$\frac{1}{2V_B} = \frac{1}{V_B} + 2 - 1$ $\frac{1}{2V_B} = \frac{1+V_B}{V_B}$ $V_B = 2V_B(1+V_B)$

$1 = 2 + 2V_B$ $2V_B = -1$ $V_B < 1$, но по смыслу задачи $V_B > 0$ не подходит

2) в 14:00 - велосипедист ост: авт.

$t_a = \frac{1}{V_a} + 2 = \frac{1}{2V_B} + 2$ $t_B = \frac{1}{V_B}$

$t_a = t_B - 1$ $\frac{1}{2V_B} + 2 = \frac{1}{V_B}$ $\frac{1+4V_B}{2V_B} = \frac{1}{V_B}$

$1 + 4V_B = 2$ $4V_B = 1$ $V_B = \frac{1}{4}$ $V_a = \frac{1}{2}$ ✓ $V_B = \frac{S}{4}$ $V_a = \frac{S}{2}$

3) в 14:00: авт, ост: авт.

$t_a = \frac{1}{V_a} + 2$ $t_B = \frac{1}{V_B}$

$t_a - 1 = t_B$ $\cancel{\frac{1}{2V_B} + 2 = \frac{1}{V_B}}$

$\frac{1}{2V_B} + 2 - 1 = \frac{1}{V_B}$ $\frac{1}{2V_B} + 1 = \frac{1}{V_B}$

$\frac{1+2V_B}{2V_B} = \frac{1}{V_B}$ $1 + 2V_B = 2$ $2V_B = 1$ $V_B = \frac{1}{2} = \frac{S}{2}$

$V_a = S$

$t_B = \frac{S}{\frac{S}{4}} = S \cdot \frac{4}{S} = 4$ часа приехали в 18:00

$\frac{S}{2V_B} + 2$ $\frac{S}{V_B}$

$\frac{S + 4V_B}{2V_B} = \frac{S}{V_B}$

$4V_B + S = 2S$

$4V_B = S$ $V_B = \frac{S}{4}$

$t_B = \frac{S}{\frac{S}{2}} = 2$

$15 + 2 = 17:00$

4) в 14:00 - авт ост: вел.

$t_A = \frac{1}{V_A}$ $t_B = \frac{1}{V_B} + 2$

$t_A - 1 = t_B$ $\frac{1}{2V_B} = \frac{1+2V_B}{V_B}$ $1 = 2 + 4V_B$ ✗

Чистовик

№2 Пусть V_A – скорость автомобилиста V_B – скорость велосипедиста

$V_A = 2V_B$. t_A – время авт. t_B – время вел. S – путь

Возможны 4 случая:

1) в 14:00 – вел. ост – вел. Тогда

$t_A = \frac{S}{V_A}$ $t_B = \frac{S}{V_B} + 2$

$t_A = t_B - 1$ (т.к. вел. ехал в итоге на час больше)

$\frac{S}{2V_B} = \frac{S}{V_B} + 1$ $\frac{S}{2V_B} = \frac{S + V_B}{V_B}$ $S = 2S + 2V_B$

$2V_B = -S$ $V_B = -\frac{S}{2} < 0$, но по смыслу задачи $V_B > 0 \Rightarrow$

$\Rightarrow$ такого не могло быть.

2) в 14:00 – вел. ост – авт. Тогда

$t_A = \frac{S}{V_A} + 2$ $t_B = \frac{S}{V_B}$ и $t_A = t_B - 1$

$\frac{S}{2V_B} + 2 = \frac{S}{V_B} - 1$ $\frac{S + 4V_B}{2V_B} = \frac{S - V_B}{V_B}$ $S + 4V_B = 2S - 2V_B$

$6V_B = S$ $V_B = \frac{S}{6}$ км/ч $V_A = \frac{S}{3}$ км/ч

$t_B = \frac{S}{\frac{S}{6}} = S \cdot \frac{6}{S} = 6$ ч. велосипедист ехал

6 часов начиная с 14:00 $\Rightarrow$ они приехали в 20:00

3) в 14:00 – авт, ост – авт. Тогда

$t_A = \frac{S}{V_A} + 2$ $t_B = \frac{S}{V_B}$

$t_A - 1 = t_B$ $\frac{S}{2V_B} + 2 - 1 = \frac{S}{V_B}$ $\frac{S}{2V_B} + 1 = \frac{S}{V_B}$ $\frac{S + 2V_B}{2V_B} = \frac{S}{V_B}$

$S + 2V_B = 2S$ $2V_B = S$ $V_B = \frac{S}{2}$ км/ч $V_A = S$ км/ч

$t_B = \frac{S}{\frac{S}{2}} = 2$ ч. Велосипедист выехал в 15:00 и

ехал 2 ч. $\Rightarrow$ они прибыли в 17:00

Чистовик

№2 4) в 14:00 - авт ост: - вел. Тогда

$t_A = \frac{S}{V_A}$ $t_B = \frac{S}{V_B} + 2$

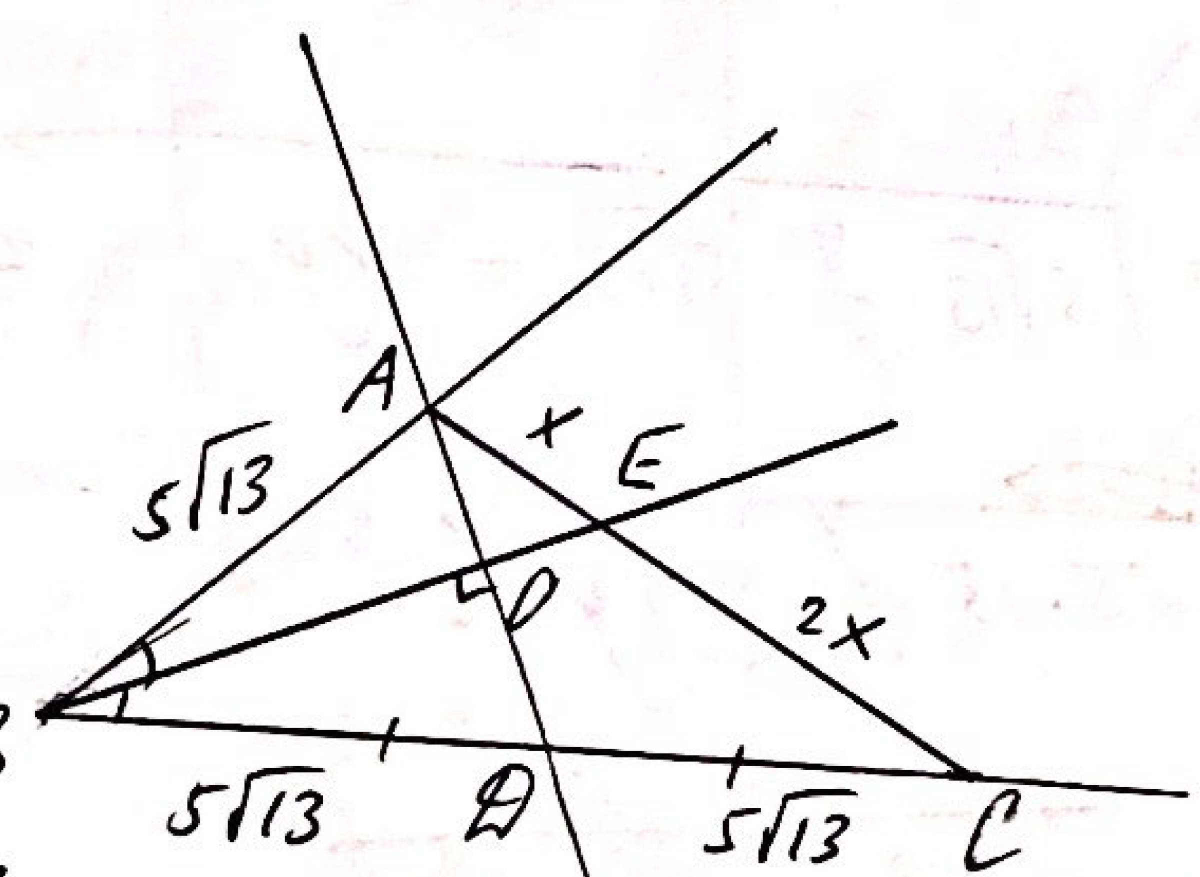
$t_A - 1 = t_B$ $\frac{S}{2V_B} - 1 = \frac{S}{V_B} + 2$ $\frac{S - 2V_B}{2V_B} = \frac{S + 2V_B}{V_B}$

$S - 2V_B = 2S + 4V_B$ $6V_B = -S$ $V_B = -\frac{S}{6} < 0$, но

по смыслу задачи $V_B > 0 \Rightarrow$ случай невозможен

Ответ: либо в 20:00, либо в 17:00

№4



1) Пусть $BE \cap AD = O$

в $\triangle BAD$: BO - выс и бисс $\Rightarrow$

$\Rightarrow \triangle BAD$ - р/б по призн

$\Rightarrow BD = BA = 5\sqrt{13}$ по опред.

AD - медиана $\Rightarrow$

$\Rightarrow DC = DB = 5\sqrt{13}$ по опред.

2) Пусть $AE = x$. Тогда по св-ву бисс. $\frac{AE}{AB} = \frac{EC}{BC}$

$EC = \frac{x}{\cancel{5\sqrt{13}}} \cdot 2 \cdot \cancel{5\sqrt{13}} = 2x$ $AC = 3x$ $BC = 2 \cdot 5\sqrt{13}$

3) по ф-ле длины бисс: $BE^2 = AB \cdot BC - AE \cdot EC$

$BE^2 = 5\sqrt{13} \cdot 2 \cdot 5\sqrt{13} - x \cdot 2x$ $BE^2 = 2 \cdot 25 \cdot 13 - 2x^2$

по ф-ле длины мед: $AD^2 = \frac{1}{4}(2AB^2 + 2AC^2 - BC^2)$

$AD^2 = \frac{1}{4}(2 \cdot 25 \cdot 13 + 2 \cdot 9x^2 - 4 \cdot 25 \cdot 13)$

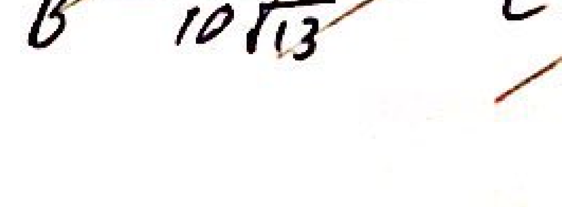
$AD = BE$ по усл $\Rightarrow AD^2 = BE^2$

$2 \cdot 25 \cdot 13 - 2x^2 = \frac{1}{4}(2 \cdot 25 \cdot 13 + 18x^2 - 4 \cdot 25 \cdot 13)$

$8 \cdot 25 \cdot 13 - 8x^2 = 18x^2 - 2 \cdot 25 \cdot 13$

~~$10 \cdot 25 \cdot 13 = 10x^2$ $x^2 = 25 \cdot 13 \Rightarrow x = 5\sqrt{13}$ $AC = 3 \cdot 5\sqrt{13}$~~

~~4)~~



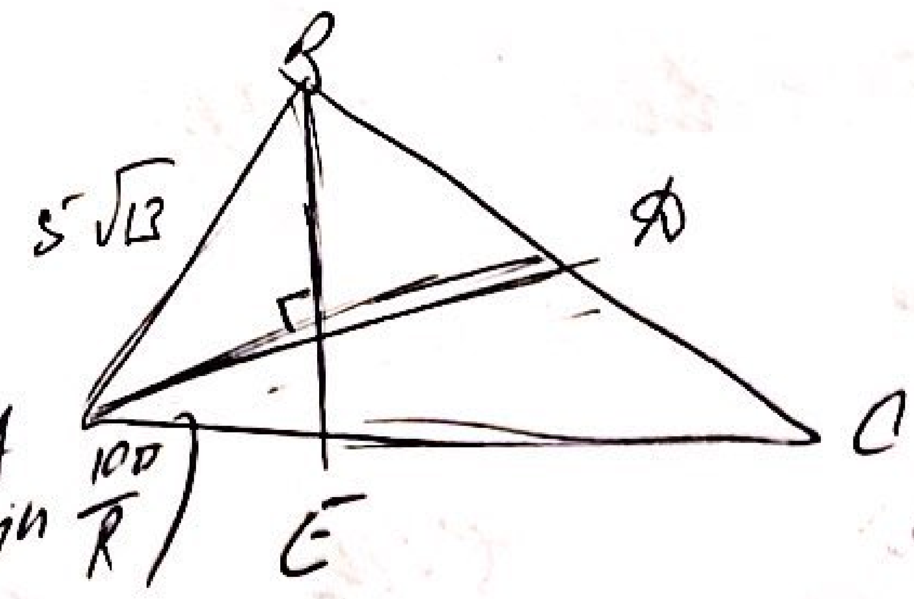
~~т. cos ∠BAC: $AB^2 = AC^2 + BC^2 - 2AC \cdot BC \cos\angle ACB$~~

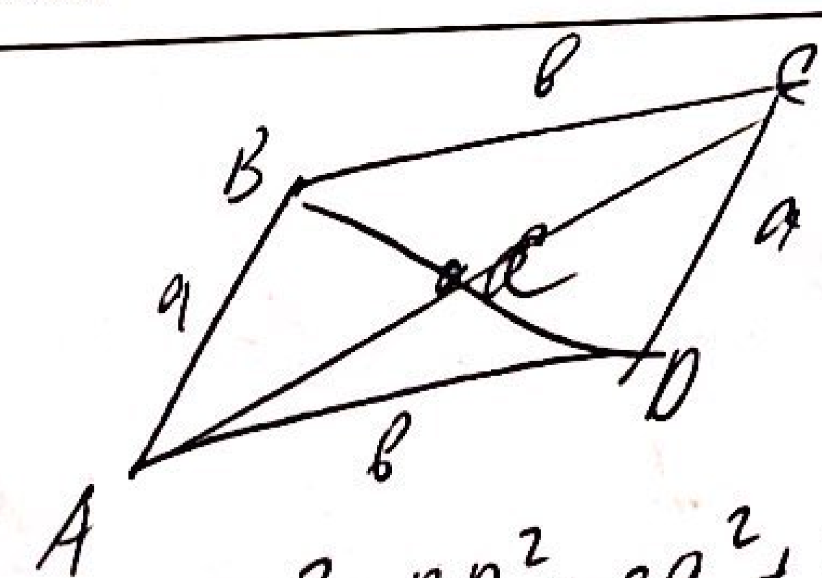
~~$25 \cdot 13 = 25 \cdot 9 \cdot 13 + 25 \cdot 4 \cdot 13 - 2 \cdot 5 \cdot 3\sqrt{13} \cdot 5 \cdot 2\sqrt{13} \cos\angle ACB$~~

~~$12 \cdot 25 \cdot 13 \cos\angle ACB = 12 \cdot 25 \cdot 13$~~

~~$\cos\angle ACB = 1$~~

Черновик





$AC^2 + BD^2 = 2a^2 + 2b^2$

$AC^2 = 2a^2 + 2b^2 - BD^2$

$4AD^2 = 2a^2 + 2b^2 - c^2$

$AD^2 = \frac{2a^2 + 2b^2 - c^2}{4}$

$4R\left(\sin\frac{6\pi}{R} + \sin\frac{8\pi}{R} + \sin\frac{10\pi}{R}\right)$

$\sin\frac{6\pi}{R} + \sin\frac{8\pi}{R} + \sin\frac{10\pi}{R} +$

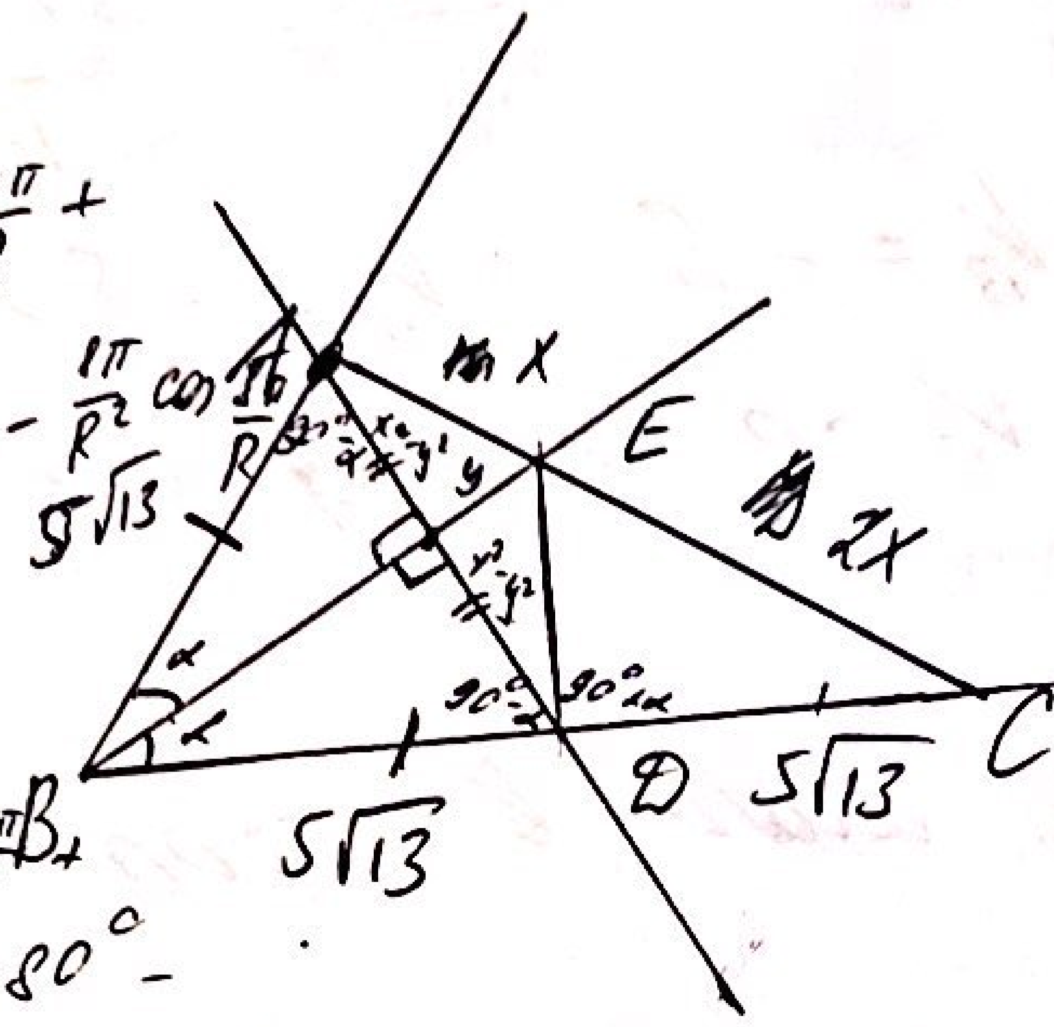
$+ R\left(-\frac{6\pi}{R^2}\cdot\cos\frac{6\pi}{R} - \frac{8\pi}{R^2}\cos\frac{8\pi}{R}\right.$

$\left.- \frac{10\pi}{R^2}\cos\frac{10\pi}{R}\right) =$

$= \sin\frac{6\pi}{R} + \sin\frac{8\pi}{R} + \sin\frac{10\pi}{R} +$

$- \frac{6\pi}{R}\cos\frac{6\pi}{R} - \frac{8\pi}{R}\cos\frac{8\pi}{R} -$

$- \frac{10\pi}{R}\cos\frac{10\pi}{R}$

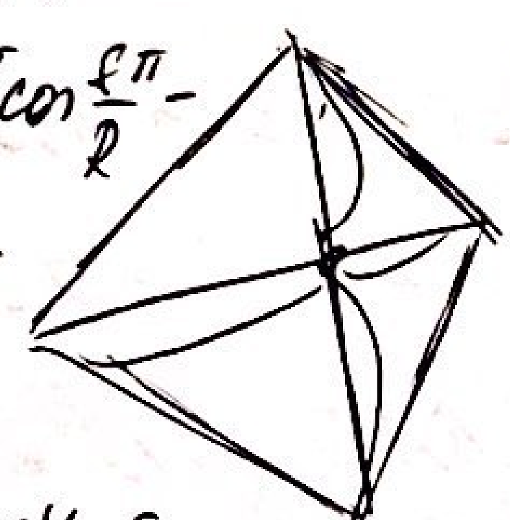


$BE = AD$

$\frac{x}{5\sqrt{13}} = \frac{y}{BC}$

$xBC = y\cdot 5\sqrt{13}$

$BC = \frac{y\cdot 5\sqrt{13}}{x}$



$BE = \sqrt{5\sqrt{13}\cdot\frac{y}{x}\cdot 5\sqrt{13} - xy} =$

$= \sqrt{25\cdot 13\frac{y}{x} - xy}$

$2(x^2 - y^2) - y =$

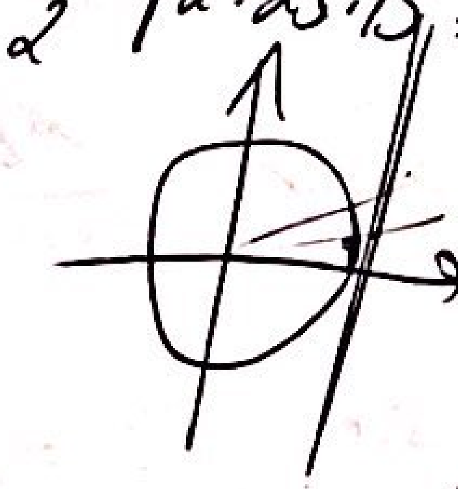
$\sin\frac{6\pi}{x} < \frac{6\pi}{x}\cos\frac{6\pi}{x}$

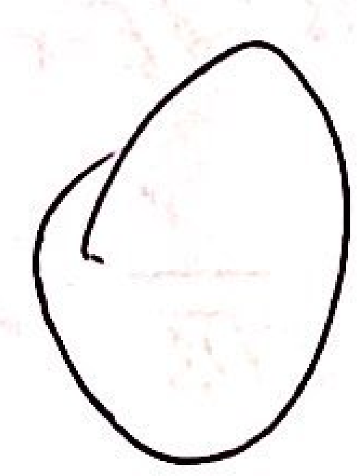
$AD = \frac{1}{2}\sqrt{2\cdot 25\cdot 13 + 2\cdot 25\cdot 13\cdot\frac{y^2}{x^2} - (x+y)^2}$

$\sin a < a\cos a$

$\operatorname{tg} a < a$

$\frac{1}{4}$





$= 25\cdot 13\frac{y}{x} - xy$

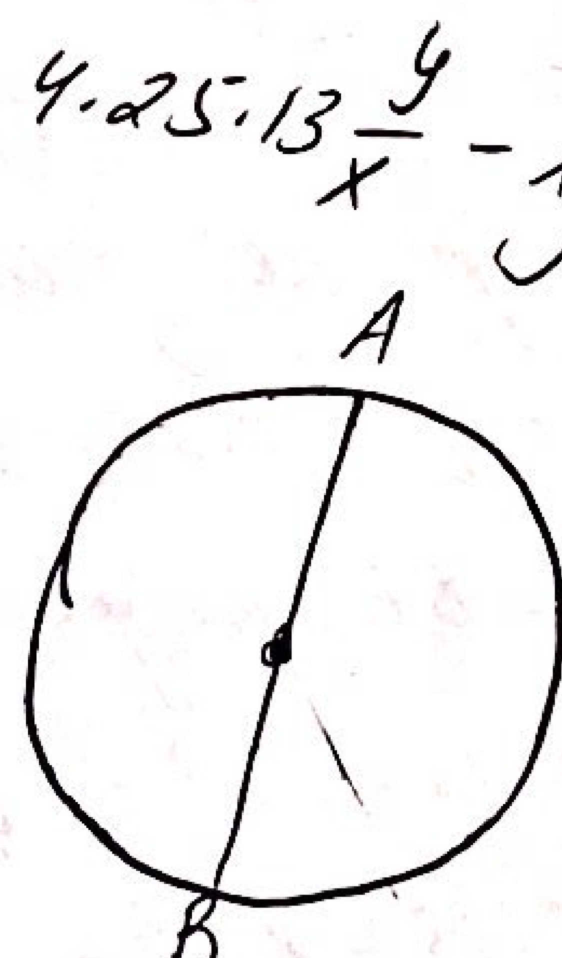
$2\cdot 25\cdot 13 + 2\cdot 25\cdot 13\frac{y^2}{x^2} - x^2 - 2xy - y^2 = 4\cdot 25\cdot 13\frac{y}{x} - xy$

$2\left(\sqrt{13} - \frac{y}{x}\right)^2 - 2$

$2\cdot 25\cdot 13\left(\frac{y^2}{x^2} - 2\frac{y}{x} + 1\right) = x^2 + xy + y^2$

$2\cdot 25\cdot 13\left(\frac{y}{x} - 1\right)^2 = x^2 + xy + y^2$

$2\cdot 5\sqrt{13}$



$16\pi = \frac{\alpha}{2\pi}$

$\alpha = \frac{16\pi}{20} = 1$

$BE^2 = 2\cdot 5\sqrt{13}\cdot 5\sqrt{13} - 2x^2 = AD^2 = \frac{1}{4}(25\cdot 13\cdot 2 + 4\cdot 25\cdot 13$

$2\cdot 25\cdot 13 - 2x^2 = \frac{1}{4}(2\cdot 25\cdot 13 + 2\cdot 9x^2 - 4\cdot 25\cdot 13)$

$8\cdot 25\cdot 13 - 8x^2 = 2\cdot 25\cdot 13 + 18x^2 - 4\cdot 25\cdot 13$

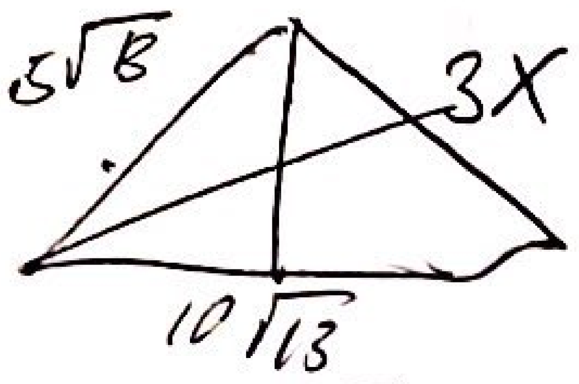
$10x^2 = 10.$

Черновик

$BE = \sqrt{ab - mn}$

$BE^2 = ab - mn = 5\sqrt{13} \cdot 2 \cdot 5\sqrt{13} - x \cdot 2x =$

$= 2 \cdot 25 \cdot 13 - 2x^2$



$5 \cdot 2\sqrt{13}$

$2 \cdot 5\sqrt{13}$

$AD^2 = \frac{1}{4}(2 \cdot 25 \cdot 13 + 29x^2 - 4 \cdot 25 \cdot 13)$

$8 \cdot 25 \cdot 13 - 8x^2 = 18x^2 - 2 \cdot 25 \cdot 13$

$13 \times 5 = 65$

$25 \times 12 = 50 + 250 = 300$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\frac{10\pi}{R} + \sin\frac{6\pi}{R} = 2\sin\frac{\frac{16\pi}{R}}{2}\cos\frac{2\pi}{R} =$

$= 2\sin\frac{8\pi}{R}\cos\frac{2\pi}{R}$

$p = 2R \cdot \sin\frac{8\pi}{R} \cdot 2\cos^2\frac{\pi}{R}$

$p = 2R\left(2\sin\frac{8\pi}{R}\cos\frac{2\pi}{R} + \sin\frac{8\pi}{R}\right)$

$f(R) = R \cdot \sin\frac{8\pi}{R}\cos^2\frac{\pi}{R}$

$p = 2R \cdot \sin\frac{8\pi}{R}\left(2\cos\frac{2\pi}{R} + 1\right)$

$3 \cdot 4 \cdot 5 \cdot 5 = 3 \cdot 100$

$\cos^2\alpha = \frac{1 + 2\cos 2\alpha}{2}$

20π

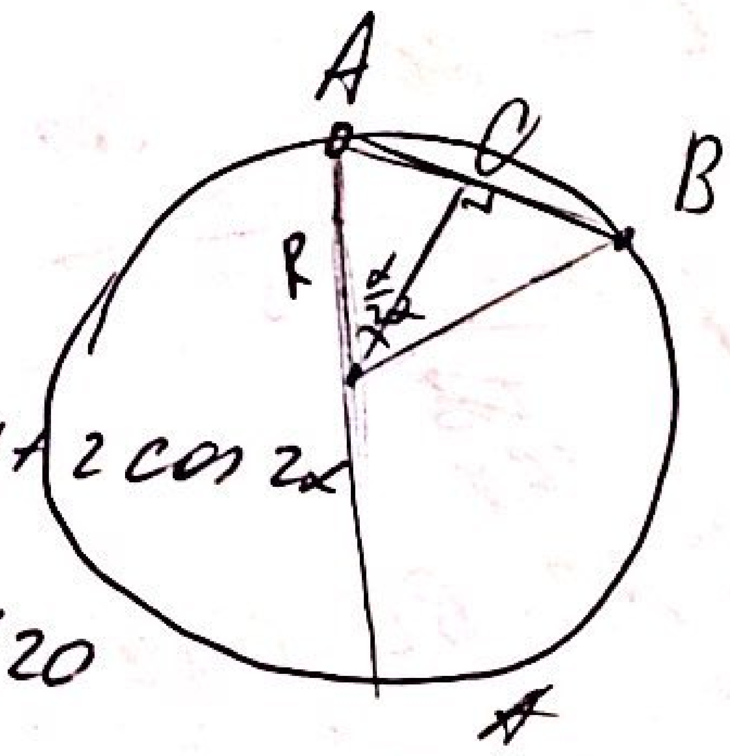
$2\cos^2\alpha = 1 + 2\cos 2\alpha$

$2\pi R \leq 40\pi$

$\pi R \leq 20\pi$

$R \leq 20$

$0 < R \leq 20$



$\frac{\alpha}{360^\circ} \cdot 2\pi R = 20\pi$

$\frac{\alpha R}{360^\circ} = 10$

$\frac{\alpha}{2\pi} \cdot 2\pi R = 20\pi$

$\alpha R = 20\pi$

$\alpha = \frac{20\pi}{R}$

$\frac{\alpha}{2} = \frac{10\pi}{R}$

$\sin\frac{\alpha}{2} = \frac{AC}{R}$

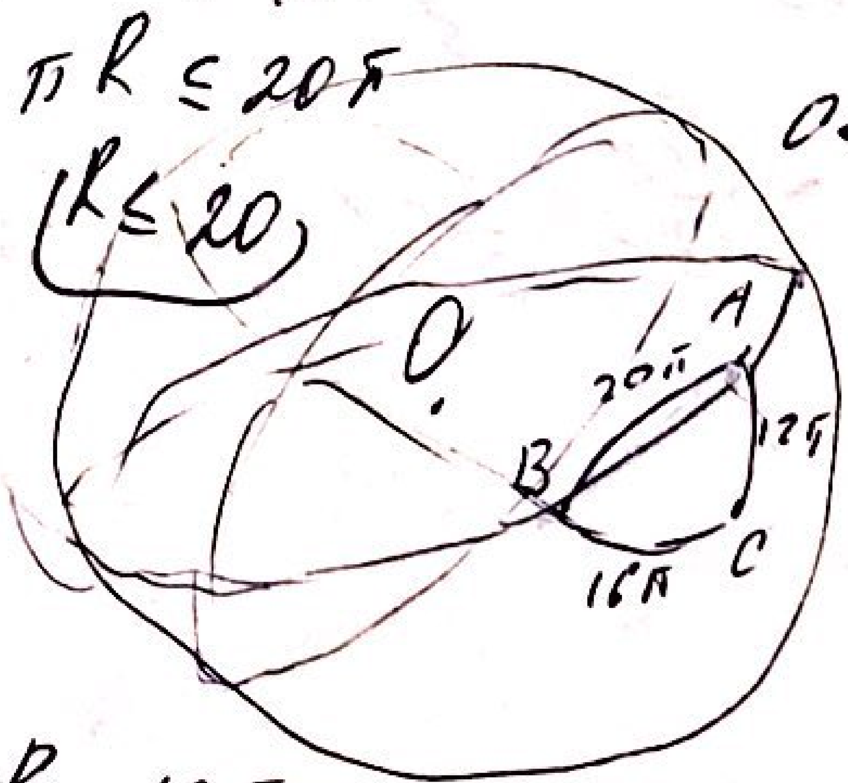
$AO = \sin\frac{R}{40\pi} \cdot R$

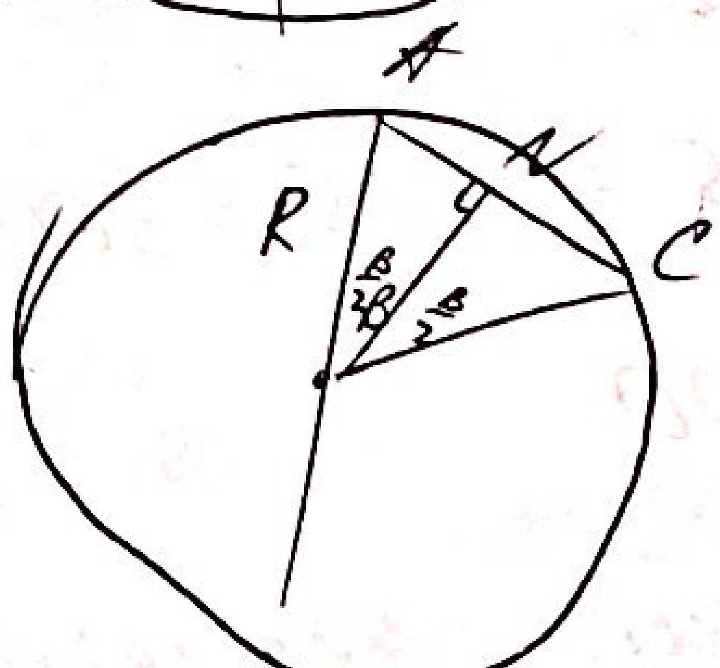
$AB = 2R \cdot \sin\frac{R}{40\pi}$

$AO = R \cdot \sin\frac{10\pi}{R}$

$AB = 2R \cdot \sin\frac{10\pi}{R}$

$f(R) =$



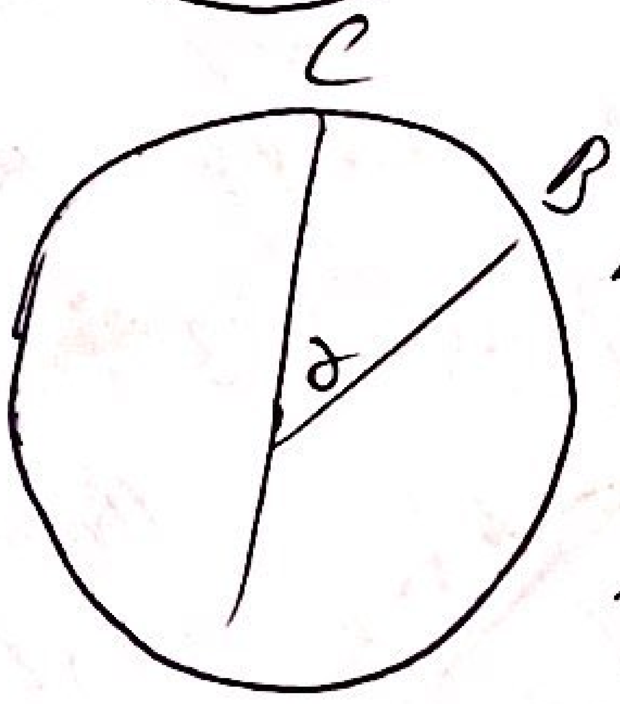


$\frac{\beta}{2\pi} \cdot 2\pi R = 16\pi$

$\beta = \frac{16\pi}{R}$ $\quad \frac{\beta}{2} = \frac{8\pi}{R}$

$AN = R \cdot \sin\frac{8\pi}{R}$ $\quad BC = 2R\sin\frac{8\pi}{R}$

$\gamma = \frac{12\pi}{R}$ $\quad \frac{\gamma}{2} = \frac{6\pi}{R}$



$AC = 2R \cdot \sin\frac{6\pi}{R}$

$p = AC + BC + AB = 2R\left(\sin\frac{6\pi}{R} + \sin\frac{8\pi}{R} + \sin\frac{10\pi}{R}\right)$

$2\pi R \geq 40\pi$

$\pi R \geq 20$

Чистовик

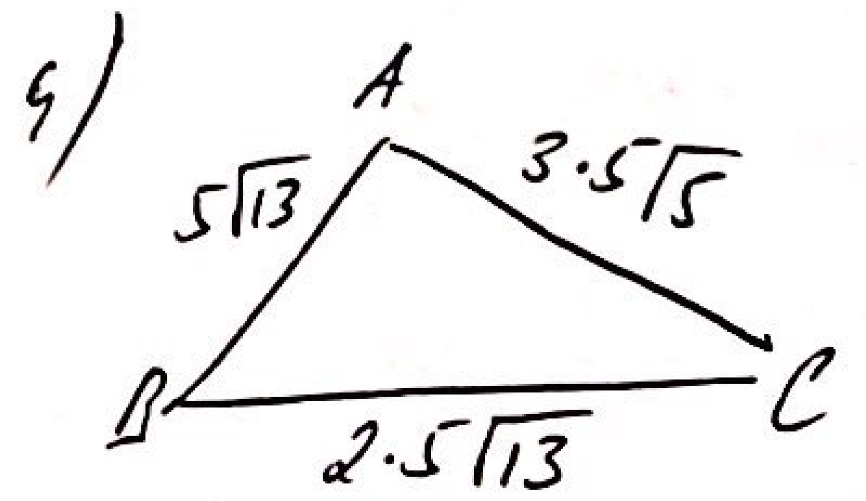
№4 $8\cdot25\cdot13-8x^2=18x^2-2\cdot25\cdot13$

$10\cdot25\cdot13=26x^2$

$x^2=\frac{\cancel{2}\cdot5\cdot25\cdot\cancel{13}}{\cancel{2}\cdot\cancel{13}}=5\cdot25$

$x=5\sqrt5$

$AC=3x=5\cdot3\sqrt5$

4) 

т. cos △ABC $AC^2=AB^2+BC^2-2\cdot AB\cdot BC\cos\angle ABC$

$9\cdot\cancel{25}\cdot5=\cancel{25}\cdot13+4\cdot\cancel{25}\cdot13-$

$-2\cdot\cancel{5}\sqrt{13}\cdot2\cdot\cancel{5}\sqrt{13}\cos\angle ABC$

$45=13+4\cdot13-4\cdot13\cos\angle ABC$

$45=5\cdot13-4\cdot13\cos\angle ABC$

$45=65-4\cdot13\cos\angle ABC$

$\cancel{4}\cdot13\cos\angle ABC=\cancel{20}\,5$ $\cos\angle ABC=\frac{5}{13}$

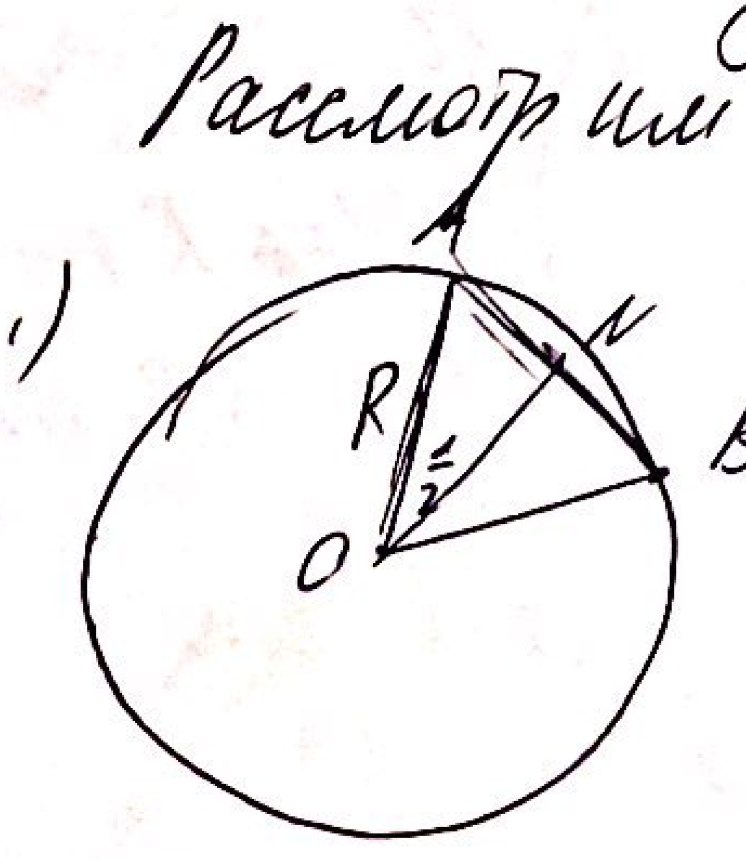
$\sin\angle ABC=\sqrt{\frac{169-25}{169}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$

5) $S_{ABC}=\frac12 AB\cdot BC\cdot\sin\angle ABC=\frac{12}{\cancel{13}}\cdot\frac{1}{\cancel{2}}\cdot5\cancel{\sqrt{13}}\cdot\cancel{2}\cdot5\cancel{\sqrt{13}}=$

$=12\cdot5\cdot5=12\cdot25=300$

Ответ: 300

№5 1) min расст. между точками — часть окружности осев. сечения между ними. Пусть R — радиус сферы. Рассмотрим 3 сечения

1) 

Пусть ∠AOB $=\alpha$, N — сер. AB △ACB — р/б ⇒

⇒ ∠ACN $=\frac{\alpha}{2}$, $AN=\frac12 AB$

$20\pi=\frac{\alpha}{2\pi}\cdot2\pi R=\alpha R$

$\alpha=\frac{20\pi}{R}$ $\frac{\alpha}{2}=\frac{10\pi}{R}$ $AN=R\sin\frac{\alpha}{2}=R\sin\frac{10\pi}{R}$

$AB=2R\sin\frac{10\pi}{R}$

 дешифр

~~Чернов~~ Черновик

$f(R) = R\sin\frac{8\pi}{R}\cos^2\frac{\pi}{R}$

$R \ge 20$

$p_3\cdot p_4\cdot p_{1876}\cdot p_{1877} = N^2$

~~$f'(R) = R'(\sin$~~

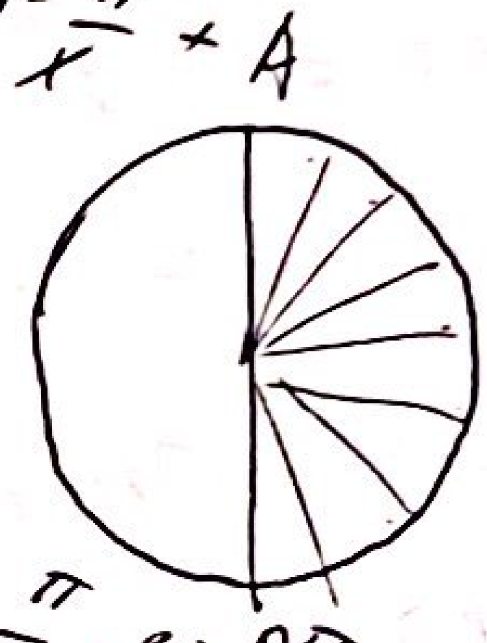
$\left(\frac{1}{x}\right)' = (x^{-1})' = -1\cdot x^{-2} = -\frac{1}{x^2}$

$f(x) = x\sin\frac{8\pi}{x}\cos^2\frac{\pi}{x}$

$f'(x) = x'\left(\sin\frac{8\pi}{x}\cos^2\frac{\pi}{x}\right) + x\left(\sin\frac{8\pi}{x}\cos^2\frac{\pi}{x}\right)' =$

$= \sin\frac{8\pi}{x}\cos^2\frac{\pi}{x} + x\left(\left(\sin\frac{8\pi}{x}\right)'\cos^2\frac{\pi}{x} + \sin\frac{8\pi}{x}\left(\cos^2\frac{\pi}{x}\right)'\right) =$

$= \sin\frac{8\pi}{x}\cos^2\frac{\pi}{x} + x\left(-\frac{8\pi}{x^2}\cdot\cos\frac{8\pi}{x}\cdot\cos^2\frac{\pi}{x} + \sin\frac{8\pi}{x}\cdot 2\cos\frac{\pi}{x}\cdot\left(-\frac{\pi}{x^2}\right)\left(-\sin\frac{\pi}{x}\right)\right) =$

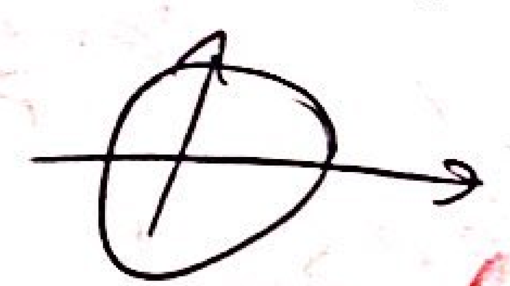


$= \sin\frac{8\pi}{x}\cos^2\frac{\pi}{x} + x\left(-\frac{8\pi}{x^2}\cos\frac{8\pi}{x}\cos^2\frac{\pi}{x} + 2\cdot\frac{\pi}{x^2}\cdot\cos\frac{\pi}{x}\cdot\sin\frac{\pi}{x}\cdot\sin\frac{8\pi}{x}\right) =$

$= \sin\frac{8\pi}{x}\cos^2\frac{\pi}{x} - \frac{8\pi}{x}\cos\frac{8\pi}{x}\cos^2\frac{\pi}{x} + \frac{2\pi}{x}\cos\frac{\pi}{x}\sin\frac{\pi}{x}\sin\frac{8\pi}{x} =$

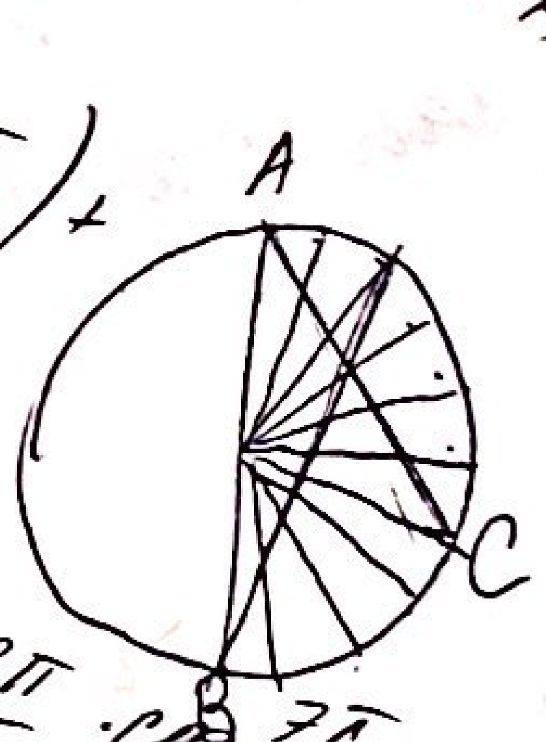
$= \overset{>0}{\cos\frac{\pi}{x}}\left(\overset{>0}{\sin\frac{8\pi}{x}}\overset{>0}{\cos\frac{\pi}{x}} - \frac{8\pi}{x}\cdot\cos\frac{8\pi}{x}\cos\frac{\pi}{x} + \frac{2\pi}{x}\sin\frac{\pi}{x}\sin\frac{8\pi}{x}\right) =$

$\frac{\overset{2}{\cancel{8}}\pi}{\underset{5}{\cancel{20}}}$



$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

$= \cos\frac{\pi}{x}\left(\frac{1}{2}\left(\sin\frac{9\pi}{x} + \sin\frac{7\pi}{x}\right) - \frac{8\pi}{x}\cdot\frac{1}{2}\left(\cos\frac{9\pi}{x} + \cos\frac{7\pi}{x}\right) + \frac{2\pi}{x}\cdot\frac{1}{2}\left(\cos\frac{9\pi}{x} - \cos\frac{7\pi}{x}\right)\right) =$



$= \cos\frac{\pi}{x}\cdot\frac{1}{2}\left(\sin\frac{9\pi}{x} + \sin\frac{7\pi}{x} - \frac{8\pi}{x}\cdot\cos\frac{9\pi}{x} + -\frac{8\pi}{x}\cdot\cos\frac{7\pi}{x} + \frac{2\pi}{x}\cdot\cos\frac{9\pi}{x} - \frac{2\pi}{x}\cdot\cos\frac{7\pi}{x}\right) =$

$= \sin\frac{9\pi}{x} + \sin\frac{7\pi}{x} - \frac{6\pi}{x}\cos\frac{9\pi}{x} - \frac{10\pi}{x}\cos\frac{7\pi}{x} \vee 0$

$\sin\frac{9\pi}{x} + \sin\frac{7\pi}{x} \vee \frac{6\pi}{x}\cdot\cos\frac{9\pi}{x} + \frac{10\pi}{x}\cos\frac{7\pi}{x}$

Чистовик

№5 2) Аналогично $BC = 2R \cdot \sin\frac{8\pi}{R}$ $AC = 2R \cdot \sin\frac{6\pi}{R}$

$P_{ABC} = AB + BC + AC = 2R\left(\sin\frac{6\pi}{R} + \sin\frac{8\pi}{R} + \sin\frac{10\pi}{R}\right)$

Рассмотрим $f(x) = \cancel{R}x\left(\sin\frac{6\pi}{x} + \sin\frac{8\pi}{x} + \sin\frac{10\pi}{x}\right)$, $x \geq 20$ ⊛

⊛ ~~10π≤πR~~ ξ.

$20\pi \leq \pi R$ (меньше половины дуги)

$\boxed{R \geq 20}$

где $f(x)$ – min, там и min P_{ABC}

$f'(x) = \sin\frac{6\pi}{x} + \sin\frac{8\pi}{x} + \sin\frac{10\pi}{x} + x\left(-\frac{6\pi}{x^2}\cos\frac{6\pi}{x} - \frac{8\pi}{x^2}\cos\frac{8\pi}{x} - \frac{10\pi}{x^2}\cos\frac{10\pi}{x}\right)$

$f'(x) = \sin\frac{6\pi}{x} - \frac{6\pi}{x}\cdot\cos\frac{6\pi}{x} + \sin\frac{8\pi}{x} - \frac{8\pi}{x}\cdot\cos\frac{8\pi}{x} + \sin\frac{10\pi}{x} - \frac{10\pi}{x}\cdot\cos\frac{10\pi}{x}$

рассмотрим отдельно $\sin\frac{6\pi}{x} - \frac{6\pi}{x}\cos\frac{6\pi}{x}$

$x \geq 20 \Rightarrow$ $0 < \frac{6\pi}{x} < \frac{\pi}{2}$ $0 < \frac{8\pi}{x} < \frac{\pi}{2}$ $0 < \frac{10\pi}{x} \leq \frac{\pi}{2} \Rightarrow$

все sin и cos > 0

$\sin\frac{6\pi}{x} \lessgtr \frac{6\pi}{x}\cos\frac{6\pi}{x}$ $\Big| : \cos\frac{6\pi}{x} > 0$

$\operatorname{tg}\frac{6\pi}{x} \lessgtr \frac{6\pi}{x}$ ($\operatorname{tg} a \geq a$, "=" при $a = 0$, а у нас $a > 0$)

$\operatorname{tg}\frac{6\pi}{x} > \frac{6\pi}{x} \Rightarrow \sin\frac{6\pi}{x} - \frac{6\pi}{x}\cos\frac{6\pi}{x} > 0$

аналогично $\sin\frac{8\pi}{x} - \frac{8\pi}{x}\cos\frac{8\pi}{x} > 0$ и

$\sin\frac{10\pi}{x} - \frac{10\pi}{x}\cos\frac{10\pi}{x} \geq 0$ $\Big\} \Rightarrow$

$f'(x) > 0$ $\forall x \geq 20 \Rightarrow f(x)$ – возрастает $\Rightarrow$

$\Rightarrow \min f(x) = f(20) \Rightarrow \min P_{ABC}$ при $R = 20$

$P = 2\cdot 20\left(\sin\frac{6\pi}{20} + \sin\frac{8\pi}{20} + \sin\frac{10\pi}{20}\right) = 40\left(\sin\frac{3\pi}{10} + \sin\frac{4\pi}{5} + 1\right)$

Ответ: $40\left(1 + \sin\frac{3\pi}{10} + \sin\frac{4\pi}{5}\right)$